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Computation of Achromatic Objectives

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Computation of Achromatic Objectives

Robert E. Stephens



National Bureau of Standards Circular 549

Issued September 20, 1954

Foreword

The National Bureau of Standards receives many requests for information on the necessary procedures for designing telescope objectives as well as for copies of designs themselves. There are very few such designs published where they are accessible to an amateur, their characteristics are rarely adequately described, and they may require glasses that are no longer obtainable. There are many amateur telescope makers who would like to compute their own designs for refracting-telescope objectives. Although there are several books in existence that describe procedures of design, most of them require the reader to review considerable theory before they get to the how-to-do-it part.

It is the purpose of this Circular to present a procedure and the necessary equations in such a way that anyone familiar with elementary algebra can apply this procedure and in a relatively short time develop the proper curves for a well corrected telescope objective. The method is also applicable to the design of low-power microscope objectives.

A. V. ASTIN, *Director*.

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Computation of Achromatic Objectives

Robert E. Stephens

Procedures for the computation of the curves for achromatic doublet objectives by the algebraic method are presented in considerable detail, using the design of two such objectives as examples. The reasons for choosing particular pairs of glass types and the changes in procedure necessary to accomplish different degrees of correction are discussed.

The information contained in this Circular is sufficient to guide the reader in computing algebraically achromatic objectives for his own specific applications. It also serves as an introduction to the algebraic method for those who may wish to delve further into the design of lens systems.

1. General Considerations

Telescope objectives usually consist of only two elementary lenses, one of crown glass and one of flint glass. The crown glass practically always faces the incident light, although equally good objectives can be made with the glasses in the reverse order. Although three or more elements are occasionally used in special purpose objectives to achieve a large relative aperture, this paper is restricted to the design of doublets, and the equations given are simplifications of the more general ones and are applicable to this special case of doublets only.

Perhaps the shortest procedure for computing telescope objectives in which the aberrations are either corrected or reduced to small prescribed values is the combination of the elegant but approximate thin-lens algebraic aberration equations with the relatively more exact, but more tedious, procedure of ray tracing. A satisfactory set of glasses and a preliminary design may be arrived at much more quickly by the algebraic method than by ray tracing. This preliminary design is then tested by ray tracing and modified slightly to achieve optimum corrections.

It is convenient to compute a design for unit focal length and scale the resulting design to any desired focal length. It is also convenient to extend the thin-lens approximation to the combination of two lenses, assuming a complete system of zero thickness. As suggested above, the designs obtained on such assumptions are not usually final but may be modified slightly by addition of thicknesses and sometimes air spaces, and by small changes in the curves. The modifications are made as a result of analysis by means of exact ray tracing.

With two elements at the disposal of the designer, an objective may be computed that has the chromatic aberration, the spherical aberration, and the coma either corrected or reduced to suitable small residuals. To accomplish such correction it is

necessary to find the dioptric powers and the shapes of the two elements.

The power of a lens is the reciprocal of its focal length, thus

$$\varphi = \frac{1}{f} = (n-1) \left(\frac{1}{r_a} - \frac{1}{r_b} \right),$$

where φ is the lens power, f the focal length, n the index of refraction of the glass, and r_a and r_b are the first and second radii of curvature, respectively. The signs of the quantities used are in accordance with the usual conventions of coordinate geometry, which is different from the convention used in some elementary physics texts but somewhat simpler to use. The light is thought of as traveling from left to right, the positive direction. A radius is measured along the axis of the lens (the line connecting the centers of curvature) from the center of the lens surface to the center of curvature. It is positive if the center of curvature is to the right of the surface, negative if to the left. In a double-convex lens the first radius is positive; the second is negative.

The above equation may also be written $\varphi = (n-1)K$, where K is called the total curvature and is defined by $K = (1/r_a) - (1/r_b)$.

If any arbitrary value be chosen for r_a , there is then a corresponding value for r_b that will give the desired value of K . Thus the shape of a lens may be changed without changing the power. Changing the shape of a lens changes the magnitudes of the spherical aberration and coma. It is possible by choosing the correct shape to reduce the coma of a simple lens to zero, but the spherical aberration can, in general, only be reduced to a minimum in this way. The following quantity, called the shape factor, is used in the equations for spherical aberration and coma: $\sigma = (r_b + r_a)/(r_b - r_a)$.

To illustrate what this means, two series of lenses whose total curvatures are the same but which have different shapes are shown in figure 1.

The following equations, which are independent of shape factors, are used to determine the powers

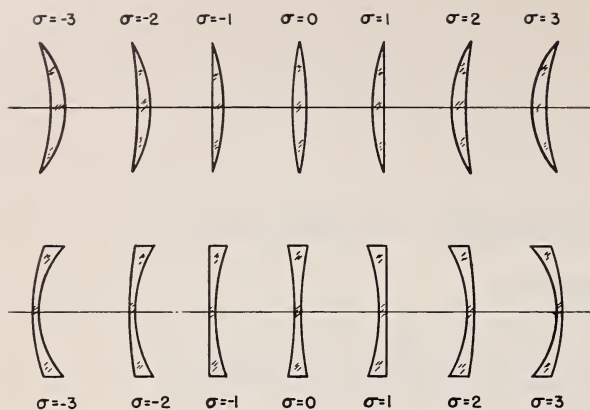


FIGURE 1. Variations in lenses with change of shape factor.

The upper series shows converging lenses, all of the same power but different shape factors. The lower series shows a similar series of diverging lenses.

of the two elements composing an objective of unit power that has a residual of chromatic aberration, abbreviated chr:

$$\varphi_1 = \frac{V_1 - V_1 V_2 \text{chr}}{V_1 - V_2}, \quad (1)$$

$$\varphi_2 = -\frac{V_2 - V_1 V_2 \text{chr}}{V_1 - V_2}, \quad (2)$$

where V is the Abbe number, $V = (n_D - 1)/n_F - n_C$; the subscripts F , D , and C refer to Fraunhofer lines of the solar spectrum whose wavelengths are 4861, 5893, and 6563 angstroms, respectively, and the subscripts 1 and 2 refer to the first and second lens elements, respectively.

The following equations are used to determine the shape factors so that the spherical aberration, abbreviated sph, and coma will have the prescribed values, usually zero:

$$\text{sph} = A_1 \varphi_1^3 + A_2 \varphi_2^3, \quad (3)$$

$$\text{coma} = C_1 \varphi_1^2 + C_2 \varphi_2^2, \quad (4)$$

where the A 's and C 's are functions of the refractive indexes, the shape factors, and the positions of image and object. They will be defined later when the need for them arises.

2. Procedure for Calculating a Design

The procedure for calculating a design will be outlined in the following by the actual calculation of an example. This example is an uncemented achromat, with full correction of spherical aberration and coma. This type of lens is adapted to use in high-power astronomical telescopes of 3-in. and greater apertures, where a cemented objective is impracticable because of the difference in the thermal expansivities of the two types of glass.

First, two suitable glass types are chosen. This is partly by experience, but almost any pair of glasses whose Abbe numbers differ by 20 or more will make a satisfactory objective. For this example, borosilicate crown, $n_e = 1.519$, $V = 64.5$, is used for the first element, and dense flint, $n_e = 1.6252$, $V = 36.2$, is used for the second element. The subscript e refers to the wavelength 5461 angstroms (mercury green). Glasses with these characteristics are made by practically all American makers of optical glass.

Next the required lens powers are computed by means of eq (1) and (2). In order to obtain full correction of the chromatic aberration we let $\text{chr} = 0$. The following powers are obtained:

$$\varphi_1 = +2.279$$

$$\varphi_2 = -1.279.$$

We next compute the positions of image and object for each element by means of the equations

$$\frac{1}{s'} = \frac{1}{s} + \varphi \quad (5)$$

and

$$s_2 = s'_1, \quad (6)$$

where s is the object distance, and s' is the image distance. We assume that the light travels from left to right. The object (or image) distance is positive if the object (or image) lies to the right of the lens, negative if to the left. For a telescope objective the object distance is taken as infinite ($1/s_1 = 0$). The resulting distances are

$$s_1 = \infty, \quad s'_1 = 0.4388$$

$$s_2 = 0.4388, \quad s'_2 = 1.000.$$

We are now ready to compute what is called the position factor,

$$\pi = \frac{s' + s}{s' - s} \quad (7)$$

or

$$\pi = -1 - \frac{2}{\varphi s}. \quad (7a)$$

For this example,

$$\pi_1 = -1$$

$$\pi_2 = +2.564.$$

The next step is the computation of the A 's and C 's for use in eqs (3) and (4). They are given by

$$A = \frac{n+2}{n(n-1)^2} \sigma^2 + \frac{4(n+1)}{n(n-1)} \sigma \pi + \frac{3n+2}{n} \pi^2 + \frac{n^2}{(n-1)^2} \quad (8)$$

$$C = \frac{n+1}{n(n-1)} \sigma + \frac{2n+1}{n} \pi. \quad (9)$$

The functions of n are computed for each of the glasses and tabulated in their order of occurrence in eqs (8) and (9). They are for borosilicate crown, $n_e=1.519$:

8.601 12.781 4.317 8.566 3.1952 2.6583;

for the dense flint, $n_e=1.6252$:

5.707 10.334 4.231 6.757 2.5837 2.6153.

The shape factors σ_1 and σ_2 remain for the present as literal quantities whose values are to be found later by algebraic solution of the aberration equations.

We now compute the A 's and C 's for the example:

$$A_1=8.601\sigma_1^2+12.781\times(-1)\sigma_1+4.317(-1)^2+8.566,$$

or

$$A_1=8.601\sigma_1^2-12.781\sigma_1+12.883;$$

$$A_2=5.707\sigma_2^2+10.334\times2.564\sigma_2+4.231\times(2.564)^2+6.757,$$

or

$$A_2=5.707\sigma_2^2+26.496\sigma_2+34.571.$$

$$C_1=3.1952\sigma_1-2.6583,$$

and

$$C_2=2.5837\sigma_2+2.6153\times2.564,$$

or

$$C_2=2.5837\sigma_2+6.706.$$

These values of the A 's and C 's are now substituted in eq (3) and (4), and as we are aiming for a fully corrected system, sph and coma are set equal to zero.

$$A_1\varphi_1^3=101.81\sigma_1^2-151.29\sigma_1+152.50$$

$$A_2\varphi_2^3=-11.940\sigma_2^2-55.435\sigma_2-72.329$$

$$\text{sph} = 101.81\sigma_1^2-151.29\sigma_1-11.940\sigma_2^2-55.435\sigma_2+80.17=0.$$

Also

$$C_1\varphi_1^2=16.595\sigma_1-13.807$$

$$C_2\varphi_2^2=4.2264\sigma_2+10.970$$

$$\text{coma}=16.595\sigma_1+4.2264\sigma_2-2.837=0.$$

These equations,

$$101.81\sigma_1^2-151.29\sigma_1-11.940\sigma_2^2-55.435\sigma_2+80.17=0$$

$$16.595\sigma_1+4.2264\sigma_2-2.837=0,$$

are to be solved simultaneously for σ_1 and σ_2 by algebraic or graphical methods. The results (obtained algebraically) are

$$\sigma_1=-0.2506 \quad \text{or} \quad 1.8225$$

$$\sigma_2=+1.6552 \quad \text{or} \quad 6.484.$$

We have now reached the solution of the problem except for converting from shape factors to radii of curvature. This conversion is accomplished by means of the equations

$$r_a=\frac{2(n-1)}{\varphi(1+\sigma)}, \quad (10)$$

$$r_b=-\frac{2(n-1)}{\varphi(1-\sigma)}. \quad (11)$$

For the example, we find

$$\begin{aligned} \text{first element} \quad & \left\{ \begin{array}{ll} r_a=+0.6078 & \text{or} \quad +0.1614, \\ r_b=-0.3642 & \text{or} \quad +0.5538; \end{array} \right. \\ \text{second element} \quad & \left\{ \begin{array}{ll} r_a=-0.3682 & \text{or} \quad -0.1306, \\ r_b=-1.4921 & \text{or} \quad -0.1783. \end{array} \right. \end{aligned}$$

These two solutions appear to be equally good to the order of approximation implied by the algebraic aberration equations. In practice, however, the second solution would never be used because the curves are so sharp that aberrations of higher orders would render it useless. The sharpness of the curves also makes production more difficult.

These designs are still for hypothetical lenses of zero thickness and separation. Before the lenses can be made, some thicknesses and separation must be allowed. It is advisable to make the center thicknesses of convex elements and the edge thicknesses of concave elements from one-eighth to one-sixth their diameters. The air space should be as small as practicable. The easiest way to decide on the thicknesses is to make a drawing of the axial section on a convenient scale. Such a drawing, for the first design only, is shown in figure 2.

It so happens that the residual aberrations of high order in the useful design—the first one—are opposite in sign to those of all orders that are introduced by the addition of thicknesses. Because of this the lens could be constructed and would perform satisfactorily for apertures up to perhaps 2

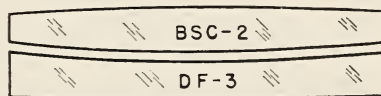


FIGURE 2. Axial section of air-spaced doublet calculated in section 2.

Appropriate thicknesses and air space have been added for a relative aperture of $f/15$.

in. if the focal length were at least 12 times the aperture.

The change in focal length on addition of thicknesses is not likely to exceed 2 percent, which is usually negligible. As the focal length of the thin-lens design is one unit, the design data for any focal length will be found by multiplying all radii, thicknesses, and spaces by the desired focal length. The design data will then be in the same kind of units as the focal length (inches, millimeters, etc.).

This type of lens has its greatest utility for objectives of 3-in. or greater apertures, where the cementing of elements is impracticable because of difference in thermal expansivity of the glasses. For small objectives, up to approximately 2-in. aperture, a cemented objective is desirable.

3. Calculation of a Cemented Doublet

To design an achromatic objective that is cementable, a further condition must be introduced. That is, that the radius of the second surface of the first element must be the same as the first radius of the second element. These surfaces must fit each other. This implies a necessary relation between the shape factors of the fitting elements. The relation also involves the lens powers and refractive indexes; it is

$$\sigma_2 = \left(\frac{n_2 - 1}{n_1 - 1} \right) \frac{\varphi_1}{\varphi_2} \sigma_1 - \left(\frac{n_2 - 1}{n_1 - 1} \right) \frac{\varphi_1}{\varphi_2} - 1. \quad (12)$$

This is quite easy to compute if one notices that the coefficient of σ_1 in the first term is the same as the second term.

The introduction of the condition for cementability makes it necessary to drop another condition, since only one shape factor is left, and it can be used to control only one aberration. We choose to retain the condition for the coma and drop that for spherical aberration. There are several considerations that lead to that choice. Coma is a more obnoxious aberration in small amounts than spherical aberration; a relatively minute amount will render the off-axis image noticeably poor. Also the solution for the coma-corrected lens is simpler than for the spherically corrected one. Further, the correction of coma practically insures that the residual spherical aberration will be a minimum (in the algebraic sense). This usually means a maximum overcorrection of spherical aberration. The magnitude of this overcorrection depends upon the types of glass chosen and can be controlled to some extent by the choice of glasses. Some overcorrection of spherical aberration is usually useful because eyepieces are always undercorrected.

The procedure for calculating the design of a cemented doublet is outlined in the following by means of an example. We will design a lens with the chromatic aberration and coma fully corrected,

which has a relatively small residual of overcorrected spherical aberration. The following glasses are to be used (again chosen as a result of experience): First element: borosilicate crown, $n_e = 1.519$, $V = 64.5$; second element: extra dense flint, $n_e = 1.6537$, $V = 33.8$.

The powers of the elements are computed, for complete achromatism, ($\text{chr} = 0$) by eq (1) and (2):

$$\varphi_1 = +2.101,$$

$$\varphi_2 = -1.101.$$

The positions of the images are computed by means of eq (5) and (6);

$$s_1 = \infty, \quad s'_1 = 0.4760,$$

$$s_2 = 0.4760, \quad s'_2 = 1.000.$$

The position factors are computed by means of eq (7) and (7a):

$$\pi_1 = -1.000,$$

$$\pi_2 = +2.817.$$

The functions of n used in computing C by eq (9) are, for the borosilicate crown, the same as in the previous example; for the flint they are 2.4548 and 2.6047.

$$C_1 = 3.1952\sigma_1 - 2.6583,$$

$$C_2 = 2.4548\sigma_2 + 7.3374.$$

But σ_2 is not independent; it is, by eq (12);

$$\sigma_2 = -2.4035\sigma_1 + 1.4035.$$

Substituting for σ_2 in C_2 , we obtain

$$C_2 = -5.9001\sigma_1 + 10.783,$$

$$C_1\varphi_1^2 = 14.104\sigma_1 - 11.734,$$

$$C_1\varphi_2^2 = -7.152\sigma_1 + 13.071,$$

$$\text{coma} = 6.952\sigma_1 + 1.337 = 0.$$

This equation determines $\sigma_1 = -0.1923$. The relation between σ_2 and σ_1 computed by eq (12) gives $\sigma_2 = +1.8657$. Equations (10) and (11) are now applied to give the radii,

$$\text{first element} \begin{cases} r_a = +0.6117 \\ r_b = -0.4144 \end{cases}$$

$$\text{second element} \begin{cases} r_a = -0.4144 \\ r_b = -1.3717. \end{cases}$$

A cross section of this design, with appropriate thicknesses added for a relative aperture of $f/5$, is shown in figure 3.



FIGURE 3. Cemented objective calculated in section 3 with appropriate thicknesses for a relative aperture of $f/5$.

We will now compute the value of the coefficient of spherical aberration. The functions of n used in computing A_2 are: 5.170 9.819 4.209 6.400.

$$A_1 = 8.601 \times (-0.1923)^2 + 12.781 \times (-0.1923) \times (-1) + 4.317 \times (-1)^2 + 8.566 = 15.659.$$

$$A_2 = 5.170 \times (1.8657)^2 + 9.819 \times 1.8657 \times 2.817 + 4.209 \times (2.817)^2 + 6.400 = 109.40;$$

$$A_1 \phi_1^3 = 145.22,$$

$$A_2 \phi_2^3 = -146.01,$$

$$\text{sph} = -0.79.$$

For a lens whose relative aperture is $f/5$ this implies (see appendix) that the marginal ray, originally parallel to the axis, after passing through the lens intersects the axis at a distance of 0.00099 unit farther from the lens than the paraxial focus (overcorrection). When appropriate thicknesses are introduced for the construction of an $f/5$ objective, it is found by exact ray tracing that this value of the spherical aberration of the marginal ray is almost exactly correct. This is a very satisfactory state of correction for use in any instrument in which the primary image needs to be well corrected, for example a telescopic rifle sight. It would also be excellent for an astronomical telescope of up to 3-in. aperture with the focal length 12 to 15 times the aperture.

4. Objectives with Residual Aberrations

Objectives for many applications should be designed to have residual aberrations to balance those remaining uncorrected in other parts of the complete system; for example, the objective of a telescope consisting of an objective and an erecting eyepiece of the type called "terrestrial." The eyepiece system of this type of telescope consists of four plano-convex lenses of the same kind of crown glass, widely spaced in an arrangement that both erects and magnifies the primary image. The powers and positions of these four elements can be so chosen that the transverse aberrations, coma, chromatic difference of magnification, and distortion are corrected. Also the image can be flattened to some extent. The longitudinal chromatic and spherical aberrations remain uncorrected. The final image may be well defined if the objective is designed to have the required amounts of these aberrations to balance those of the eyepiece.

In practice the eyepiece would be designed first. Its longitudinal chromatic and spherical aberrations would be evaluated. The objective would then be computed by exactly the same procedure as in the examples, using appropriate values for chr and sph in eq (2) and (3), instead of zero. Formulas for relating chr and sph to the longitudinal values of the corresponding aberrations will be found in the appendix.

The air-spaced doublet can be designed to have almost any overcorrection or undercorrection desired with almost any choice of crown and flint glass. The design of a cemented doublet with optimum residual aberrations is somewhat more difficult because of the necessity for satisfying the condition for cementing, eq (13). It can usually be accomplished, however, by a suitable choice of glasses. This entails making a complete design with a choice of glasses based upon experience or even just a guess. The residual sph is then computed. If it is not close enough to the desired value, another design is then made with one of the glasses changed. The experience acquired by these successive trials will guide the designer in further trials, which should quickly lead to a satisfactory design.

It is in problems like this one that the algebraic method shows up to best advantage, as compared with more exact but much slower methods.

5. Final Correction

When a preliminary design that seems suitable has been found by the algebraic method, it is always desirable to test its characteristics, after suitable thicknesses have been added, by means of exact ray tracing. It is beyond the scope of this Circular to treat ray tracing methods in detail but some mention of the procedure for achieving optimum correction is in order.

A procedure, which seems not to have been published previously, that has been used with considerable success by the author is presented in this paragraph. Suitable thicknesses and spaces are given to the design obtained by means of the algebraic aberration equations. The performance of the design is then analyzed by means of ray tracing and the residual aberrations evaluated. The residual aberrations will probably not have the desired values. To obtain a design that will be closer to the desired one the quantities,

$$\Delta \text{chr} = (\text{chr}_{TL} - \text{chr}),$$

$$\Delta \text{sph} = (\text{sph}_{TL} - \text{sph}),$$

$$\Delta \text{coma} = (\text{coma}_{TL} - \text{coma}),$$

are computed. In these the quantities with the subscript TL are those that were used in the algebraic computation of the design; those without subscripts are those found by exact ray tracing. The quantities Δchr , Δsph , and Δcoma are then

added to the corresponding chr_{TL} , sph_{TL} , and coma_{TL} that were used previously in the algebraic design. A new design is then found by solving eq (1), (2), (3), and (4) with these new values for chr , sph , and coma . On adding thicknesses and testing by ray tracing it will usually be found that the second design is entirely satisfactory. If in some unfavorable case it is still too far from optimum, this procedure may be repeated with benefit.

6. Appendix

6.1. Symbols

A =function of index of refraction, position factor, and shape factor used in calculation of spherical aberration. It is defined by eq (8).

C =function of index of refraction, position factor, and shape factor used in calculation of coma. It is defined by eq (9). Used also as a subscript referring to homogeneous light whose wavelength is 6563 angstroms, the red spectrum line of hydrogen.

D =used as a subscript referring to homogeneous light whose wavelength is 5893 angstroms, the yellow spectrum lines of sodium.

e =used as a subscript referring to homogeneous light whose wavelength is 5461 angstroms, the green spectrum line of mercury.

F =used as a subscript referring to homogeneous light whose wavelength is 4861 angstroms, the blue spectrum line of hydrogen.

f =focal length.

K =total curvature of a lens.

n =index of refraction.

p =semiaperture of a lens.

r_a =radius of curvature of the incidence side of a lens.

r_b =radius of curvature of the emergence side of a lens.

s =distance from a lens to the object (measured along the axis).

s' =distance from a lens to the image (measured along the axis).

V =the Abbe number, which is a measure of the dispersive properties of a glass. (If V is large, the dispersion is low.)

β =field angle or angle between the axis and an oblique incident ray.

π =used in this circular as the position factor used in the calculation of spherical aberration and coma. It is defined by eq (7) and (7a).

σ =the shape factor of a lens element. It is defined by

$$\sigma = \frac{r_b + r_a}{r_b - r_a}$$

ϕ =the power of a lens. It is the reciprocal of the focal length.

6.2. Design equations¹ in the order of application

For determining the lens powers:

$$\phi_1 = \frac{V_1 - V_1 V_2 \text{ chr}}{V_1 - V_2} \quad (1)$$

$$\phi_2 = -\frac{V_2 - V_1 V_2 \text{ chr}}{V_1 - V_2} \quad (2)$$

For computing the positions of the objects and images:

$$\frac{1}{s'} = \frac{1}{s} + \phi \quad (5)$$

¹ Numbered as in the text.

$$s_2 = s_1 \quad (6)$$

The position factor

$$\pi = \frac{s' + s}{s' - s} \quad (7)$$

or

$$\pi = -1 - \frac{2}{\phi s} \quad (7a)$$

(when $s = \infty$, $\pi = -1$).

$$A = \frac{n+2}{n(n-1)^2} \sigma^2 + \frac{4(n+1)}{n(n-1)} \sigma \pi + \frac{3n+2}{n} \pi^2 + \frac{n^2}{(n-1)^2} \quad (8)$$

$$C = \frac{n+1}{n(n-1)} \sigma + \frac{2n+1}{n} \pi \quad (9)$$

Spherical aberration

$$\text{sph} = A_1 \phi_1^3 + A_2 \phi_2^3 \quad (3)$$

Coma

$$\text{coma} = C_1 \phi_1^2 + C_2 \phi_2^2 \quad (4)$$

Condition for cementability,

$$\sigma_2 = \frac{(n_2-1)}{(n_1-1)} \frac{\phi_1}{\phi_2} \sigma_1 - \frac{(n_2-1)}{(n_1-1)} \frac{\phi_1}{\phi_2} - 1 \quad (12)$$

The radii

$$r_a = \frac{2(n-1)}{\phi(1+\sigma)} \quad (10)$$

$$r_b = \frac{2(n-1)}{\phi(1-\sigma)} \quad (11)$$

6.3. Explanation of Aberrations

Longitudinal chromatic aberration is the distance, measured along the axis from the position of the blue paraxial image (usually F) to the red image (usually C). It is for a lens of unit focal length exactly equal to the chr of eq (1) and (2).

Longitudinal spherical aberration is the distance measured along the axis from the focus for marginal rays to the focus for paraxial rays. Marginal rays, in this case, are defined as those rays, initially parallel to the axis, entering the lens at a distance p from the center. This is illustrated in figure 4.

For a lens of unit focal length the algebraic approximation to the longitudinal spherical aberration is $\frac{1}{2} p^2 \text{sph}$, where p is the semiaperture.

Coma is rather difficult to define except when spherical aberration is absent. In this case it is the failure of the marginal rays to come to the same focal point as the central rays. This is because of a transverse displacement and applies only to the image of a point off the axis. If the necessary rays were traced by an exact method a measure of coma is shown in figure 5.

The meridional coma is related to the coma of eq (4) by the equation, meridional coma = $\frac{1}{2} p^2 \text{coma} \tan \beta$, where p

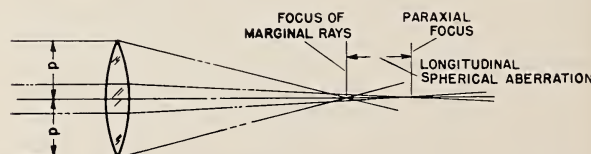


FIGURE 4. Longitudinal spherical aberration.

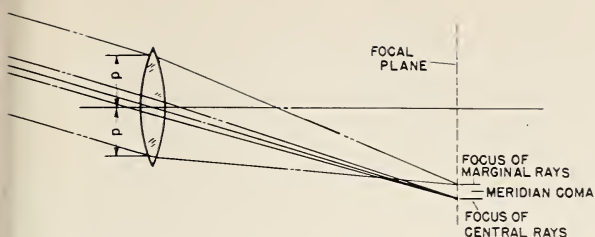


FIGURE 5. Meridional coma (spherical aberration corrected).

is the semiaperture and β the angle of obliquity of the incident rays.

A quantity more often used, because it can be computed from the data of the rays shown in figure 4 without tracing oblique rays, is the offense against the sine condition invented by Conrady (see appendix), and abbreviated OSC'. The algebraic approximation to OSC' is, $OSC' = \frac{1}{4}p^2$ coma. The comatic flare is inward (toward center of the field) if coma (eq 4) is positive.

6.4. References

The following is a list of references that contain information useful to students of lens design. It is far from a complete bibliography on the subject but contains those works that the author has selected for their excellence in the particular field.

A. E. Conrady, *Applied optics and optical design* (Oxford University Press, New York, N. Y., London, 1929; also an American reprint, 1943). This is a 518-page book. It is probably the best introduction to the theory and methods of lens design available. It contains the theoretical derivations of all the equations used and also detailed descriptions, with examples, of procedures of lens designing. It includes ray-tracing methods and an introduction to the algebraic method and to combination methods. The practical worker may consider it too theoretical, but it is in reality a very practical book.

B. K. Johnson, *Optical design and computation* (The Hatton Press, Ltd., London, 1948). This book describes, in considerable detail, procedures, ray tracing and algebraic, that may be used in designing several different types of lenses, for example, telescope and microscope objectives, eyepieces, and simple photographic objectives. It makes use of the equations of Conrady without giving theoretical background. It is shorter and less expensive than the book by Conrady.

D. H. Jacobs, *Fundamentals of optical engineering* (McGraw-Hill Book Co., New York, N. Y., 1944). This book contains a section on elementary methods of lens design applied to simple objectives and eyepieces, mostly following Conrady.

I. C. Gardner, *Applications of the algebraic aberration equations to optical design*, Sci. Papers Bureau of Standards 22 (1927-28), S 550. This is out of print but may be consulted in some libraries. This booklet contains a very good introduction to the algebraic method of designing lenses. Equations and procedures are given whereby preliminary designs of more complex systems of several elements widely separated may be quickly obtained. The procedures described depend upon graphical solutions to a considerable extent.

Robert E. Stephens, *The design of triplet anastigmat lenses of the Taylor type*, Journal of the Optical Society of

America 38, 1032 (December 1948). This paper gives the algebraic aberration equations used in the synthesis of systems in general form, applicable to systems of any number of elements with any number of air spaces. The specialized application of these to the design of systems of three elements with two air spaces is described in detail. An outline of the procedure for final adjustment of the system by means of ray tracing is given. The procedures described may be modified by the reader so as to be applicable to systems of greater or less complexity.

The following books are excellent college-grade textbooks on the principles of optics, especially those applicable to optical instruments. They contain very little material on design methods or procedures.

Hardy and Perrin, *The principles of optics* (McGraw-Hill Book Co., New York, N. Y., 1932).

L. C. Martin, *Technical optics*, vol. I (Sir Isaac Pitman & Sons, Ltd., New York, N. Y., and London).

6.5. Optimally Corrected Objective

The design worked out as an example in section 2 has been slightly modified by means of ray tracing, to achieve optimum corrections. The final specifications for a lens of unit focal length and a clear aperture of 0.06667 ($f/15$) are:

First element	Borosilicate crown glass, $n_D=1.517$, $n_e=1.519$, $V=64.5$
	First radius = +0.6110 (convex)
	Second radius = -0.3580 (convex)
	Axial thickness = 0.00826.
Second element	Dense flint glass, $n_D=1.621$, $n_e=1.6252$, $V=36.2$
	First radius = -0.3594 (concave)
	Second radius = -1.4327 (convex)
	Axial thickness = 0.00703.

The axial separation between the two elements is 0.00198. The distance from the fourth surface to the focus is 0.98838.

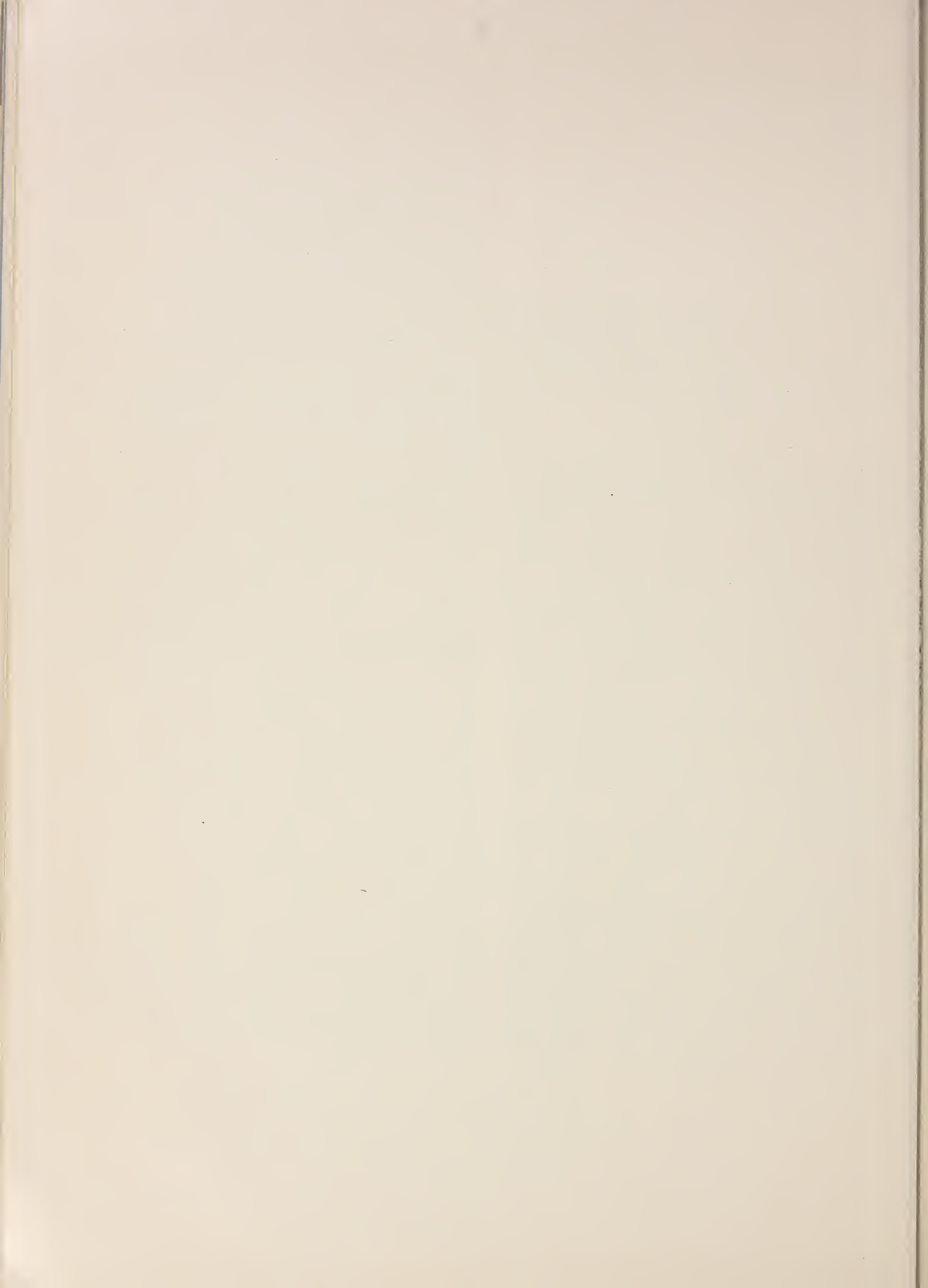
This objective is corrected for spherical aberration, primary chromatic aberration, and coma. The distance from the focus for e light to the common focus of C and F lights is 0.000617 (secondary chromatic aberration). The zonal spherical aberration and the chromatic variation of spherical aberration are negligible.

This design may be scaled to any desired size by multiplying all the radii, thicknesses, and the space by a scale factor. Its performance would be excellent, certainly up to 12-in. aperture and 15 ft focal length.

The scale factor to be used would be found as follows: For an objective of aperture x , factor = $x \div 0.06667$, for example, for a 4-in. objective, focal length = 60 in., factor = $4 \text{ in.} \div 0.06667 = 60 \text{ in.}$

It will be noticed by comparison that the modifications in the radii are quite small. The original algebraic design would probably perform almost as well as the modified one. Nevertheless, in constructing the lens one should aim for the specifications of the modified design because the tolerance for deviations in construction will be greater and symmetrically distributed.





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